Chapter R
Relativity

Conceptual Problems

3 • If event A occurs at a different location than event B in some reference frame, might it be possible for there to be a second reference frame in which they occur at the same location? If so, give an example. If not, explain why not.

Determine the Concept Yes. Let the initial frame of reference be frame 1. In frame 1 let *L* be the distance between the events, let *T* be the time between the events, and let the +*x* direction be the direction of event B relative to event A. Next, calculate the value of *L*/*T*. If *L*/*T* is less than *c*, then consider the two events in a reference frame 2, a frame moving at speed $v = L/T$ in the +*x* direction. In frame 2 both events occur at the same location.

5 • Two events are simultaneous in a frame in which they also occur at the same location. Are they simultaneous in all other reference frames?

Determine the Concept Yes. If two events occur at the same time *and* place in one reference frame they occur at the same time *and* place in all reference frames. (Any pair of events that occur at the same time *and* at the same place in one reference frame are called a spacetime coincidence.)

11 •• Many nuclei of atoms are unstable; for example, ^{14}C, an isotope of carbon, has a half-life of 5700 years. (By definition, the *half-life* is the time it takes for any given number of unstable particles to decay to half that number of particles.) This fact is used extensively for archeological and biological dating of old artifacts. Such unstable nuclei decay into several decay products, each with significant kinetic energy. Which of the following is true? (*a*) The mass of the unstable nucleus is larger than the sum of the masses of the decay products. (*b*) The mass of the unstable nucleus is smaller than the sum of the masses of the decay products. (*c*) The mass of the unstable nucleus is the same as the sum of the masses of the decay products. Explain your choice.

Determine the Concept Because mass is converted into the kinetic energy of the fragments, the mass of the unstable nucleus is larger than the sum of the masses of the decay products. $\boxed{(a)}$ is correct.

Estimation and Approximation

13 •• In 1975, an airplane carrying an atomic clock flew back and forth at low altitude for 15 hours at an average speed of 140 m/s as part of a time-dilation experiment. The time on the clock was compared to the time on an atomic clock kept on the ground. What is the time difference between the atomic clock on the airplane and the atomic clock on the ground? (Ignore any effects that accelerations of the airplane have on the atomic clock that is on the airplane. Also assume that the airplane travels at constant speed.)

Picture the Problem We can use the time dilation equation to relate the elapsed time in the frame of reference of the airborne clock to the elapsed time in the frame of reference of the clock kept on the ground.

Use the time dilation equation to relate the elapsed time Δt according to the clock on the ground to the elapsed time Δt_0 according to the airborne atomic clock:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \qquad (1)$$

Because $v << c$, we can use the approximation $\frac{1}{\sqrt{1-x}} \approx 1+\frac{1}{2}x$ to obtain:

$$\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \approx 1+\frac{1}{2}\left(\frac{v}{c}\right)^2$$

Substitute in equation (1) to obtain:

$$\Delta t = \left[1+\frac{1}{2}\left(\frac{v}{c}\right)^2\right]\Delta t_0 = \Delta t_0 + \frac{1}{2}\left(\frac{v}{c}\right)^2 \Delta t_0 \qquad (2)$$

where the second term represents the additional time measured by the clock on the ground.

Evaluate the proper elapsed time according to the clock on the airplane:

$$\Delta t_0 = (15\,\text{h})\left(3600\,\frac{\text{s}}{\text{h}}\right) = 5.40\times10^4\ \text{s}$$

Substitute numerical values and evaluate the second term in equation (2):

$$\Delta t' = \frac{1}{2}\left(\frac{140\,\text{m/s}}{2.998\times10^8\ \text{m/s}}\right)^2 \left(5.40\times10^4\ \text{s}\right) = 5.89\times10^{-9}\ \text{s} \approx \boxed{5.9\,\text{ns}}$$

Length Contraction and Time Dilation

17 • In the reference frame of a pion in Problem 18, how far does the laboratory travel in 2.6×10^{-8} s?

Picture the Problem We can use $\Delta x = v\Delta t_\pi$, where Δt_π is the proper mean lifetime of the pions, to find the distance traveled by the laboratory frame in a typical pion lifetime.

The average distance the laboratory will travel before the pions decay is the product of the speed of the pions and their proper mean lifetime:

$$\Delta x = v\Delta t_{\pi}$$

Substitute numerical values and evaluate Δx:

$$\Delta x = (0.85c)(2.6\times10^{-8}\text{ s}) = 6.63\text{ m}$$
$$= \boxed{6.6\text{ m}}$$

21 • A spaceship travels from Earth to a star 95 light-years away at a speed of 2.2×10^8 m/s. How long does the spaceship take to get to the star (*a*) as measured on Earth and (*b*) as measured by a passenger on the spaceship?

Picture the Problem We can use $\Delta x = v\Delta t$ to find the time for the trip as measured on Earth and $\Delta t_0 = \Delta t\sqrt{1-(v/c)^2}$ to find the time measured by a passenger on the spaceship.

(*a*) Express the elapsed time, as measured on Earth, in terms of the distance traveled and the speed of the spaceship:

$$\Delta t = \frac{\Delta x}{v}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{95c\cdot\text{y}}{2.2\times10^8\text{ m/s}}\times\frac{9.461\times10^{15}\text{ m}}{c\cdot\text{y}}$$
$$= 4.09\times10^9\text{ s}\times\frac{1\text{ y}}{31.56\text{ Ms}} = 129\text{ y}$$
$$= \boxed{1.3\times10^2\text{ y}}$$

(*b*) A passenger on the spaceship will measure the proper time:

$$\Delta t_0 = \Delta t\sqrt{1-\left(\frac{v}{c}\right)^2}$$

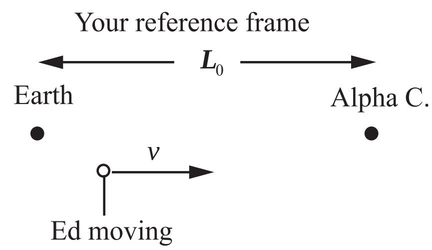
Substitute numerical values and evaluate the proper time:

$$\Delta t_0 = (129\text{ y})\sqrt{1-\frac{(2.2\times10^8\text{ m/s})^2}{(2.998\times10^8\text{ m/s})^2}}$$
$$= \boxed{88\text{ y}}$$

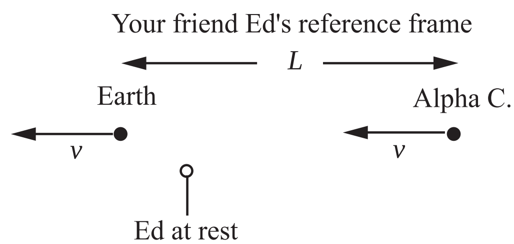
25 •• Your friend, who is the same age as you, travels to the star Alpha Centauri, which is 4.0 light-years away, and returns immediately. He claims that the entire trip took just 6.0 y. What was his speed? Ignore any accelerations of your friend's spaceship and assume the spaceship traveled at the same speed during the entire trip.

Picture the Problem To calculate the speed in the reference frame of the friend, who is named Ed, we consider each leg of the trip separately. Consider an imaginary stick extending from Earth to Alpha Centauri that is at rest relative to Earth. In Ed's frame the length of the stick, and thus the distance between Earth and Alpha Centauri, is shortened in accord with the length contraction formula. As three years pass on Ed's watch Alpha Centauri travels at speed v from its initial location to him.

Sketch the situation as it is in your reference frame. The distance between Earth and Alpha C. is the rest length L_0 of the stick discussed in Picture the Problem:



Sketch the situation as it is in Ed's reference frame. The distance between Earth and Alpha C. is the length L of the moving stick discussed in **Picture the Problem**:



The two events are Ed leaves Earth and Ed arrives at Alpha Centauri. In Ed's frame these two events occur at the same place (next to Ed). Thus, the time between those two events Δt_0 is the proper time between the two events.

$$\Delta t_0 = 3\text{ y}$$

The distance L traveled by Alpha Centauri in Ed's frame during the first three years equals the speed multiplied by the time in Ed's frame:

$$L = v\Delta t_0$$

The distance between Earth and Alpha Centauri in Ed's frame is the contracted length of the imaginary stick:

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

Equate these expressions for L to obtain:

$$L_0\sqrt{1-\frac{v^2}{c^2}} = v\Delta t_0$$

Substituting numerical values yields:

$$(4.0\,c\cdot\text{y})\sqrt{1-\frac{v^2}{c^2}}=v(3.0\text{ y})$$

or

$$1-\frac{v^2}{c^2}=\frac{9}{16}\frac{v^2}{c^2}\Rightarrow\frac{25}{16}\left(\frac{v^2}{c^2}\right)=1$$

Solving for v gives:

$$v=\boxed{0.80c}$$

The Relativity of Simultaneity

35 •• In an inertial reference frame S, event B occurs 2.00 μs after event A and 1.50 km distant from event A. How fast must an observer be moving along the line joining the two events so that the two events occur simultaneously? For an observer traveling fast enough is it possible for event B to precede event A?

Picture the Problem Because event A is ahead of event B by $L_\text{p}v/c^2$ where v is the speed of the observer moving along the line joining the two events, we can use this expression and the given time between the events to find v.

Express the time Δt between events A and B in terms of L_p, v, and c:

$$\Delta t=\frac{L_\text{p}v}{c^2}\Rightarrow v=\frac{c^2\Delta t}{L_\text{p}}$$

Substitute numerical values and evaluate v:

$$v=\frac{(2.998\times10^8\text{ m/s})^2(2.00\,\mu\text{s})}{1.50\,\text{km}}$$

$$=(1.20\times10^8\text{ m/s})\left(\frac{c}{2.998\times10^8\text{ m/s}}\right)$$

$$=\boxed{0.400c}$$

Rewrite the expression for v in terms of t_A and t_B yields:

$$v=\frac{c^2(t_\text{B}-t_\text{A})}{L_\text{p}}$$

Express the condition on $t_\text{B}-t_\text{A}$ if event B is to precede event A:

$$t_\text{B}-t_\text{A}<0\text{ and }v>\frac{c^2(t_\text{B}-t_\text{A})}{L_\text{p}}$$

Substitute numerical values and evaluate v:

$$v>\frac{(2.998\times10^8\text{ m/s})^2(2.00\,\mu\text{s})}{1.50\,\text{km}}$$

$$=1.20\times10^8\text{ m/s}=0.400c$$

Event B can precede event A provided $v > 0.400c$.

Relativistic Energy and Momentum

41 • How much energy would be required to accelerate a particle of mass m from rest to (*a*) $0.500c$, (*b*) $0.900c$, and (*c*) $0.990c$? Express your answers as multiples of the rest energy, mc^2.

Picture the Problem We can use Equation R-14 to find the energy required to accelerate this particle from rest to the given speeds.

From Equation R-14 we have:

$$K = \frac{mc^2}{\sqrt{1-(v/c)^2}} - mc^2 = \left(\frac{1}{\sqrt{1-(v/c)^2}} - 1\right)mc^2$$

(*a*)Substitute numerical values and evaluate $K(0.500c)$:

$$K(0.500c) = \left(\frac{1}{\sqrt{1-(0.500c/c)^2}} - 1\right)mc^2 = \boxed{0.155mc^2}$$

(*b*) Substitute numerical values and evaluate $K(0.900c)$:

$$K(0.900c) = \left(\frac{1}{\sqrt{1-(0.900c/c)^2}} - 1\right)mc^2 = \boxed{1.29mc^2}$$

(*c*) Substitute numerical values and evaluate $K(0.990c)$:

$$K(0.990c) = \left(\frac{1}{\sqrt{1-(0.990c/c)^2}} - 1\right)mc^2 = \boxed{6.09mc^2}$$

45 •• (*a*) Show that the speed v of a particle of mass m and total energy E is given by $\frac{v}{c} = \left[1 - \frac{(mc^2)^2}{E^2}\right]^{1/2}$ and that when E is much greater than mc^2, this can be approximated by $\frac{v}{c} \approx 1 - \frac{(mc^2)^2}{2E^2}$. Find the speed of an electron with kinetic energy of (*b*) 0.510 MeV and (*c*) 10.0 MeV.

Picture the Problem We can solve the equation for the relativistic energy of a particle to obtain the first result and then use the binomial expansion subject to $E >> mc^2$ to obtain the second result. In Parts (*b*) and (*c*) we can use the first expression obtained in (*a*), with $E = E_0 + K$, to find the speeds of electrons with the given kinetic energies. See Table 39-1 for the rest energy of an electron.

(*a*) The relativistic energy of a particle is given by Equation R-15:

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

Solving for v/c yields:

$$\frac{v}{c} = \boxed{\left[1-\frac{\left(mc^2\right)^2}{E^2}\right]^{1/2}} \qquad (1)$$

Expand the radical expression binomially to obtain:

$$\frac{v}{c} = \sqrt{1-\frac{\left(mc^2\right)^2}{E^2}} = 1-\frac{1}{2}\frac{\left(mc^2\right)^2}{E^2} + \text{higher-order terms}$$

Because the higher-order terms are much smaller than the 2nd-degree term when $E >> mc^2$:

$$\frac{v}{c} \approx \boxed{1-\frac{\left(mc^2\right)^2}{2E^2}}$$

(*b*) Solve equation (1) for v:

$$v = c\sqrt{1-\frac{\left(mc^2\right)^2}{E^2}}$$

Because $E = E_0 + K$:

$$v = c\sqrt{1-\frac{E_0^2}{\left(E_0+K\right)^2}} = c\sqrt{1-\frac{1}{\left(1+\frac{K}{E_0}\right)^2}}$$

For an electron whose kinetic energy is 0.510 MeV:

$$v(0.510\,\text{MeV}) = c\sqrt{1-\frac{1}{\left(1+\frac{0.510\,\text{MeV}}{0.511\,\text{MeV}}\right)^2}} = \boxed{0.866c}$$

(*c*) For an electron whose kinetic energy is 10.0 MeV:

$$v(10.0\,\text{MeV}) = c\sqrt{1 - \frac{1}{\left(1 + \dfrac{10.0\,\text{MeV}}{0.511\,\text{MeV}}\right)^2}} = \boxed{0.999c}$$

General Problems

53 •• Particles called muons traveling at 0.99995*c* are detected at the surface of Earth. One of your fellow students claims that the muons might have originated from the Sun. Prove him wrong. (The proper mean lifetime of the muon is 2.20 μs.)

Picture the Problem Your fellow student is thinking that the time dilation factor might allow muons to travel the 150,000,000,000 m from the Sun to Earth. You can discredit your classmate's assertion by considering the mean lifetime of the muon from Earth's reference frame. Doing so will demonstrate that the distance traveled during as many as 5 proper mean lifetimes is consistent with the origination of muons within Earth's atmosphere.

The distance, in the Earth frame of reference, a muon can travel in *n* mean lifetimes τ is given by:

$$d = n\frac{d_0}{\sqrt{1-(v/c)^2}} = \frac{nv\tau}{\sqrt{1-(v/c)^2}} = \frac{n(v/c)c\tau}{\sqrt{1-(v/c)^2}}$$

Substitute numerical values for *v*, *c*, and τ and simplify to obtain:

$$d = n\frac{(0.99995)(2.998\times10^8\text{ m/s})(2.20\,\mu\text{s})}{\sqrt{1-(0.99995)^2}} = (66.0\text{ km})n$$

In 5 lifetimes a muon would travel a distance:

$d = (66.0\text{ km})(5) = 330\text{ km}$, a distance approximating a low-Earth orbit.

In 100 lifetimes, $d \approx 6600$ km, or approximately one Earth radius. This relatively short distance should convince your classmate that the origin of the muons that are observed on Earth is within our atmosphere and that they certainly are not from the Sun.